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THE CYCLOID TRAJECTORY METHOD FOR WRAPING SURFACES ASSOCIATED OF ROLLING CENTROID STUDY.* II APPLICATIONS.

Drd. ing. Virgil TEODOR
Prof. dr. ing. Nicolae OANCEA
Universitatea "Dunărea de Jos" din Galati

ABSTRACT

The paper shows some examples for the applications of the trajectory method theorem for gear rack, gear-shaped cutter and rotary cutter tools profiling. It's shown some numerical examples and graphical illustration of a trajectory family envelope, for simples profiles generating of the vortex surfaces edged semi-products.

1. Introduction.

It was shown [3] a new way for the explaining the envelope of the conjugate profile process – the cycloid trajectory principle, which for was enunciated and demonstrated a specify theorem order to determining the enveloping for a profile associated with a rolling centroids pair.
Also, it was shown the specifics algorithms for known process: gear rack, gear-shaped cutter and rotary cutter engendering, as envelope engendering proceedings with linear contacts (the case of the surfaces associated with a rolling axoids pair).
Is evident, in the case of the planes profiles in wrapping examinations, that the wrapping

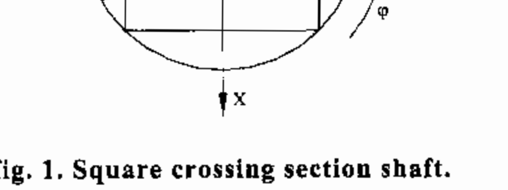

fig. 1. Square crossing section shaft.

Correlated with the algorithm it's shown the crossing section of shaft —the profile vortex for engendering Σ— and reference systems:
xy is the fixe system, solidary with semi-product axe;

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profiles contact is punctiform. Next, will be shown a number of applications of the new expression form for the wrapping surfaces (profiles) fundamental theorem and for the specific engendering process –the cycloid trajectory method.

2. Gear rack engendering.

Next we will present an applications ensemble for gear-rack engendering based of this theorem.

2.1. The gear-rack tool profiling for the square profile generating.

First application, linked also with figure 1, it referring to gear-rack tool profiling for the engendering of a square crossing section shaft.
XY is the mobile system, solidary with the profile vortex for engendering;
ξη is the mobile system, solidary with gear-rack tool;
It's defined parametrical equations of the shaft flank (the Σ profile of the ordered vortex of profiles):

$$M\begin{vmatrix} X=-a; \\ Y=u, \end{vmatrix} \qquad (1)$$

with u variable parameter.
From the relative motion of the associated spaces of the centroids

$$\xi=\omega_3^T(\varphi)\cdot X-\begin{Vmatrix} -Rr \\ -Rr\cdot\varphi \end{Vmatrix} \qquad (2)$$

can determine the trajectory family:

$$(T_\Sigma)_\varphi\begin{vmatrix} \xi=-a\cdot\cos\varphi-u\cdot\sin\varphi+Rr; \\ \eta=-a\cdot\sin\varphi+u\cdot\cos\varphi+Rr\cdot\varphi. \end{vmatrix} \qquad (3)$$

The trajectory family enveloping (3), it's obtaining, as was shown in [3], associating of the $(T_\Sigma)_\varphi$ family the enveloping condition in form

$$\frac{\xi'_u}{\xi'_\varphi}=\frac{\eta'_u}{\eta'_\varphi} \qquad (4)$$

which, for the concrete case, become

$$\frac{-\sin\varphi}{a\cdot\sin\varphi-u\cdot\cos\varphi}= =\frac{\cos\varphi}{-a\cdot\cos\varphi-u\cdot\sin\varphi+Rr}. \qquad (5)$$

The (3) and (5) equation ensemble represent the gear-rack profile mutually enveloping for the shaft having the crossing section a square.

In figure 2 and table 1, it's shown the form and coordinates of the gear-rack profile, also the trajectory form $(T_\Sigma)_\varphi$ of the points belongs of the Σ profile in connection with the reference system associated with the C_2 centroid of the gear-rack tool, in case Re=28,28 mm; a=20 mm.

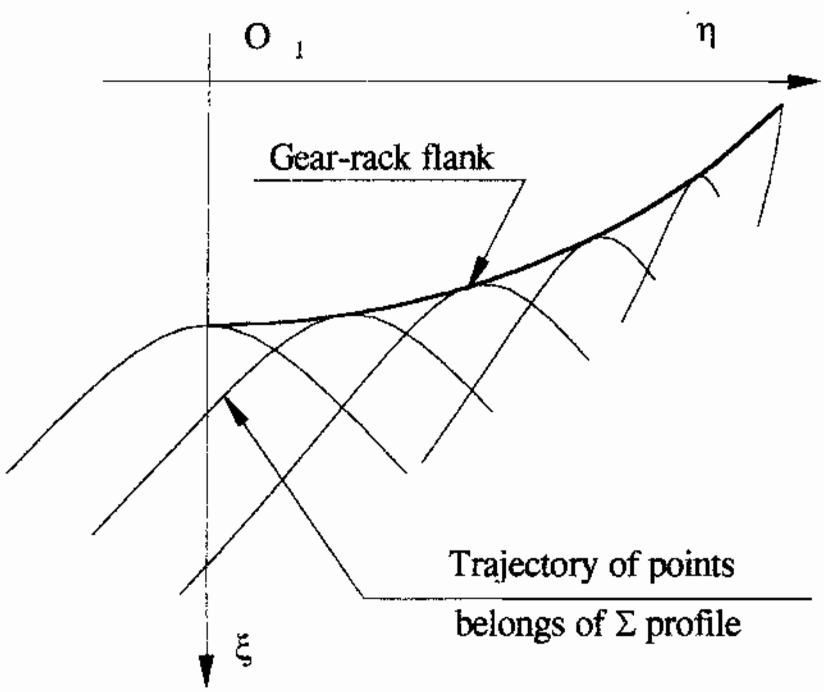


fig. 2. Trajectory of the semi-product points and the enveloping for the gear-rack profile.

Table 1.

Nr. crt.	ξ [mm]	η [mm]	u [mm]	φ [rad]
1	14.2741	0.99160	0.70000	0.02040
10	13.8133	6.73940	4.78000	0.13980
20	13.3494	9.46140	6.74000	0.19790
30	12.9168	11.4046	8.16000	0.24030
40	12.4503	13.1591	9.46000	0.27960
50	11.9442	14.8042	10.7000	0.31730
60	11.4359	16.2662	11.8200	0.35190
70	10.9412	17.5509	12.8200	0.38330
80	10.3879	18.8608	13.8600	0.41630
90	9.83130	20.0684	14.8400	0.44770
100	9.25260	21.2270	15.8000	0.47910
110	8.52610	22.5638	16.9400	0.51680
120	7.71480	23.9264	18.1400	0.55760
130	6.97660	25.0642	19.1800	0.59380
136	6.38820	25.9099	19.9800	0.62200

2.2. Gear-rack tool for spline shaft engendering.

In figure 3, it's shown the crossing section of the spline shaft and the reference systems (see paragraph 2.1).

The parametric equation of Σ flank have the form:

$$\Sigma\begin{vmatrix} X=-u; \\ Y=b, \end{vmatrix} \qquad (6)$$

with u variable parameter.

In (2) motion, the trajectory family $(T_\Sigma)_\varphi$ it is generated:

$$(T_\Sigma)_\varphi\begin{vmatrix} \xi=-u\cdot\cos\varphi-b\cdot\sin\varphi+Rr; \\ \eta=-u\cdot\sin\varphi+b\cdot\cos\varphi+Rr\cdot\varphi \end{vmatrix} \qquad (7)$$

For this concretely case, the enveloping condition (4) can bented in form:

$$\frac{-\cos\varphi}{u\cdot\sin\varphi - b\cdot\cos\varphi} = \frac{-\sin\varphi}{-u\cdot\cos\varphi - b\cdot\sin\varphi + Rr}. \qquad (8)$$

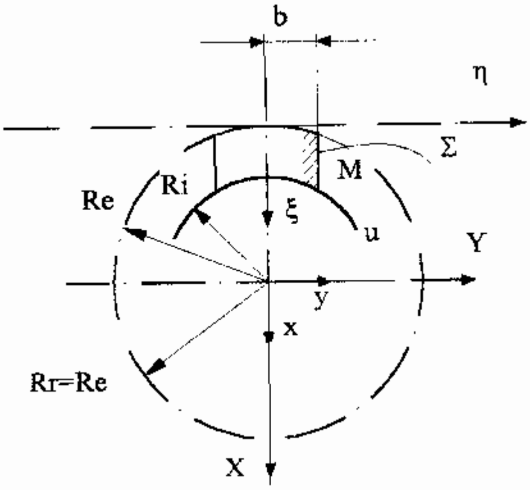


fig. 3. Spline shaft.

The (7) and (8) equation ensemble represent the gear-rack tool profile mutually enveloping of the spline shaft flank.

The (7) equations can be interpreted as trajectory of points belongs of Σ profile given the reference system of tool.

In figure 4 and table 2, it's shown the $(T_\Sigma)_\varphi$ trajectory (7) and the envelope —the gear-rack flank, in case: Ri=24 mm, Re=27 mm, b=4 mm.

Table 2.

Nr. crt.	ξ [mm]	η [mm]	u [mm]	Φ [rad]
1	3.5472	5.2020	23.6643	0.3814
2	3 3431	5.1639	24.2643	0.4400
5	3.1621	5.0787	24.4643	0.4400
6	2.9396	4.9760	24.5643	0.4200
10	2.7590	4.8944	24.7643	0.4200
11	2.5400	4.8016	24.8643	0.4000
15	2.1471	4.6407	25.1643	0.3800
16	1.94610	4.5636	25.2643	0.3600
20	1.20430	4.3075	25.7643	0.3000
21	0.94140	4.2288	25.9643	0.2800
25	0.02480	4.0144	26.7021	0.1058

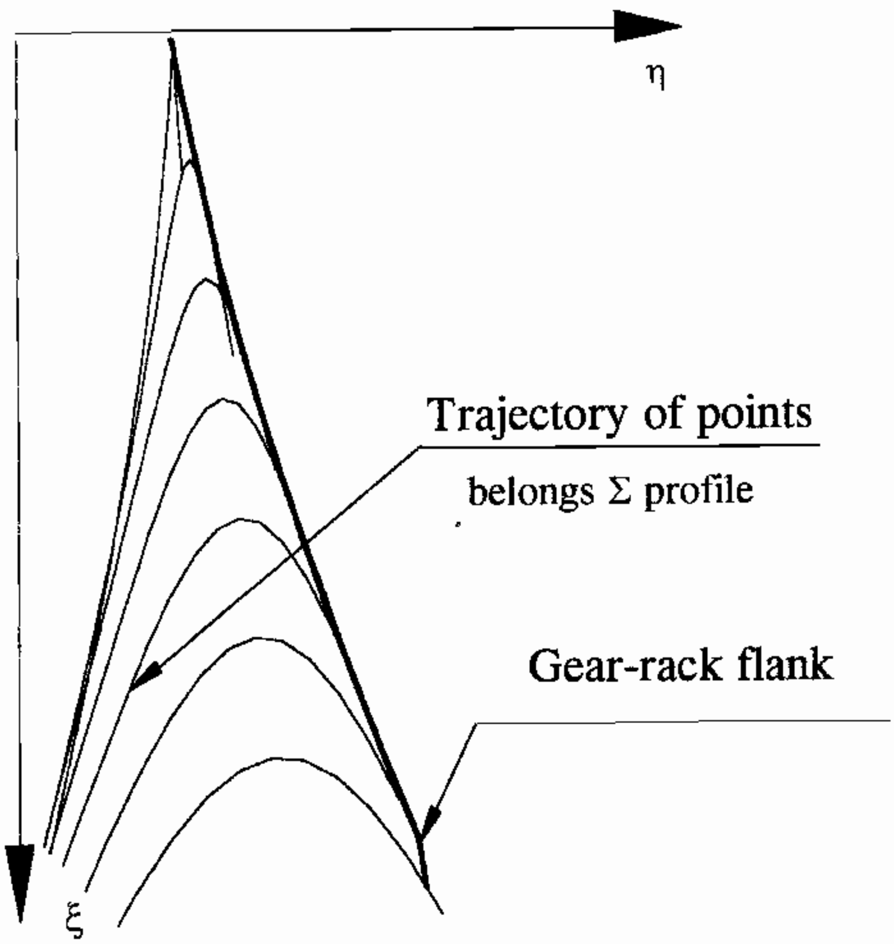


fig. 4. The $(T_\Sigma)_\varphi$ trajectory family.

3. Gear-shaped cutter engendering.

3.1. Gear-shaped cutter for a square bushing.

Next, it’s shown, an application for a gear-shaped cutter profiling for a square bushing engendering, see figure 5.

XY is the mobile system, solidary with the bushing (semi-product).

ξη is the mobile system, solidary with the gear-shaped cutter.

The centroids wrapping pair is formed by circles having Rrp and Rrs radius.

It’s defined parametrical equations of the Σ profile of the vortex (figure 5).

$$\Sigma : X = -a; Y = u, \qquad (9)$$

with u variable parameter.

From relative motion

$$\xi = \omega_2(\varphi_2)\cdot\left[\omega_3^T(\varphi_1)\cdot X - a\right] \qquad (10)$$

with

$$a = \begin{Vmatrix} -A_{12} \\ 0 \end{Vmatrix}; A_{12} = Rrp - Rrs \qquad (11)$$

after substitutions,

$$\begin{Vmatrix} \xi \\ \eta \end{Vmatrix} = \begin{Vmatrix} \cos\varphi_2 & \sin\varphi_2 \\ -\sin\varphi_2 & \cos\varphi_2 \end{Vmatrix} \cdot \left[\begin{Vmatrix} \cos\varphi_1 & -\sin\varphi_1 \\ \sin\varphi_1 & \cos\varphi_1 \end{Vmatrix} \cdot \begin{Vmatrix} -a \\ u \end{Vmatrix} - \begin{Vmatrix} -A_{12} \\ 0 \end{Vmatrix} \right] \qquad (12)$$

result the $(T_\Sigma)_\varphi$ trajectory family of points belong of Σ profile in relative motion of this given the system ξη:

$$(T_\Sigma)_{\varphi 1} \begin{vmatrix} \xi = -a\cdot\cos(\varphi_1 - \varphi_2) - \\ -u\cdot\sin(\varphi_1 - \varphi_2) + A_{12}\cdot\cos\varphi_2; \\ \eta = -a\cdot\sin(\varphi_1 - \varphi_2) + \\ +u\cdot\cos(\varphi_1 - \varphi_2) - A_{12}\cdot\sin\varphi_2. \end{vmatrix} \qquad (13)$$

or, with notation,

$$\varphi_2 = i\cdot\varphi_1, i = \frac{Rrp}{Rrs} \qquad (14)$$

$$(T_\Sigma)_{\varphi 1} \begin{vmatrix} \xi = -a\cdot\cos(1-i)\varphi_1 - \\ -u\cdot\sin(1-i)\varphi_1 + A_{12}\cdot\cos(i\varphi_1); \\ \eta = -a\cdot\sin(1-i)\varphi_1 + \\ +u\cdot\cos(1-i)\varphi_1 - A_{12}\cdot\sin(i\varphi_1). \end{vmatrix} \qquad (15)$$

According to the theorem of the cycloid trajectories, the enveloping of this trajectories family, is determined associating the condition (4) of those equations, where:

$$\begin{aligned} &\xi'_u = -\sin(1-i)\varphi_1; \\ &\eta'_u = \cos(1-i)\varphi_1; \\ &\xi'_{\varphi 1} = a(1-i)\sin(1-i)\varphi_1 - \\ &-u(1-i)\cos(1-i)\varphi_1 - iA_{12}\sin(i\varphi_1); \\ &\eta'_{\varphi 1} = -a(1-i)\cos(1-i)\varphi_1 - \\ &-u(1-i)\sin(1-i)\varphi_1 - iA_{12}\cos(i\varphi_1). \end{aligned} \qquad (16)$$

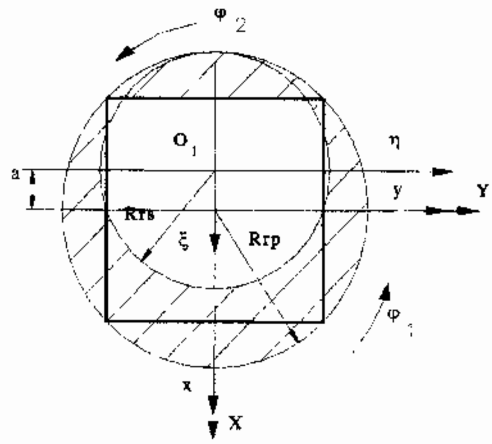


fig. 5. The square bushing profile.

In table 3 and figure 6, it’s shown the coordinates and profile form for gear-shaped cutter and the $(T_\Sigma)_\varphi$ trajectories which have the seeking profile as enveloping, for conditions a=40; Rrs=30 mm; i=3/4.

Table 3.

Nr. crt.	ξ [mm]	η [mm]	u [mm]	Φ [rad]
1	-12.9295	0.00000	0.0000	0.0000
2	-12.9259	0.74000	0.8200	0.0290
5	-12.8545	3.38540	3.7500	0.1330
6	-12.7970	4.49740	4.9800	0.1770
10	-12.4974	8.09770	8.9500	0.3220
11	-12.3891	9.04510	9.9900	0.3610
15	-11.8505	12.7053	13.980	0.5170
16	-11.7209	13.4273	14.760	0.5490
20	-11.6028	14.0496	15.430	0.5770
21	-11.5900	14.1147	15.500	0.5800
25	-11.5028	14.5527	15.970	0.6000
26	-11.4895	14.6181	16.040	0.6030
30	-11.4043	15.0294	16.480	0.6220
31	-11.3405	15.3293	16.800	0.6360
35	-10.8491	17.4396	19.030	0.7380
36	-10.6073	18.3714	20.000	0.7854

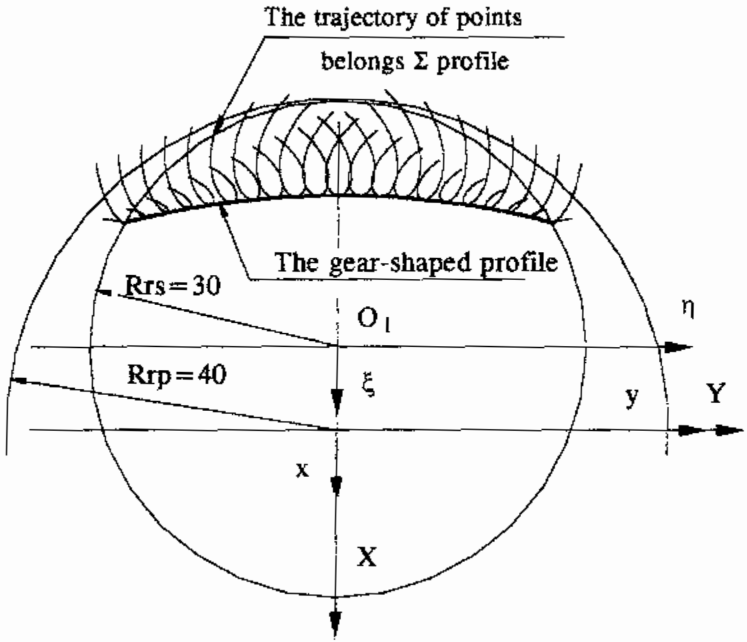


fig. 6. The gear-shaped cutter.

3.2. Gear-shaped cutter for internal chamfered groove.

In figure 7, it's shown the transversal profile of the grooves, the reference systems and the rolling centroids.

The grooves flank, Σ, have the equations:

$$\Sigma\begin{cases} X = -R_0 - u \cdot \cos\varepsilon; \\ Y = u \cdot \sin\varepsilon, \end{cases} \qquad (17)$$

with u as variable parameter.

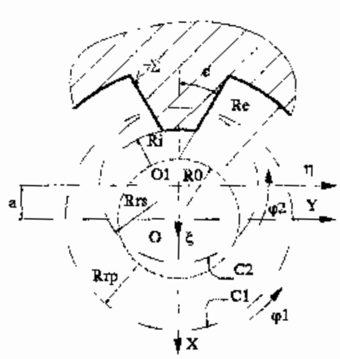


fig. 7. Internal chamfered grooves.

In relative motion (10), the Σ profile coordinates (17) determine the trajectory family

$$(T_\Sigma)_{\varphi 1}\begin{cases} \xi = (-R_0 - u \cdot \cos\varepsilon) \cdot \cos(1-i)\varphi_1 - \\ - u \cdot \sin\varepsilon \cdot \sin(1-i)\varphi_1 + \\ + A_{12} \cdot \cos(i\varphi_1); \\ \eta = (-R_0 - u \cdot \cos\varepsilon) \cdot \sin(1-i)\varphi_1 + \\ + u \cdot \sin\varepsilon \cdot \cos(1-i)\varphi_1 - \\ - A_{12} \cdot \sin(i\varphi_1). \end{cases} \qquad (18)$$

The enveloping condition (4) presume the partial derived calculation:

$$\begin{aligned} \xi'_u &= -\cos\varepsilon \cdot \cos(1-i)\varphi_1 - \\ &- \sin\varepsilon \cdot \sin(1-i)\varphi_1; \\ \eta'_u &= -\cos\varepsilon \cdot \sin(1-i)\varphi_1 + \\ &+ \sin\varepsilon \cdot \cos(1-i)\varphi_1; \end{aligned} \qquad (19)$$

$$\begin{aligned} &\xi'_{\varphi 1} = -(R_0 - u \cdot \cos\varepsilon)(1-i)\sin(1-i)\varphi_1 - \\ &- u(1-i)\sin\varepsilon\cos(1-i)\varphi_1 - \\ &- iA_{12}\sin(i\varphi_1); \\ &\eta'_{\varphi 1} = (R_0 - u\cos\varepsilon)(1-i)\cos(1-i)\varphi_1 - \\ &- u(1-i)\sin\varepsilon\sin(1-i)\varphi_1 - \\ &- iA_{12}\cos(i\varphi_1). \end{aligned} \qquad (20)$$

In figure 8 and table 4, it's shown the (18) trajectory form, the enveloping profile (the gear-shaped cutter profile) and its coordinates.

Table 4.

Nr. crt.	ξ [mm]	η [mm]	u [mm]	φ [mm]
1	-20.4454	3.2467	5.6420	-0.1531
5	-20.6638	3.3538	6.0020	-0.1331
10	-21.0093	3.5249	6.5020	-0.1121
15	-21.2932	3.6692	6.9020	-0.0961
20	-21.6440	3.8533	7.4020	-0.0741
25	-21.9971	4.0447	7.9020	-0.0511
30	-22.2825	4.2040	8.3020	-0.0321
35	-22.4972	4.3269	8.6020	-0.0171
40	-22.7882	4.4971	9.0020	0.00290
45	-23.0810	4.6740	9.4020	0.02390
50	-23.4508	4.9037	9.9020	0.05390
55	-23.8301	5.1486	10.402	0.08590
60	-24.2217	5.4113	10.902	0.12290

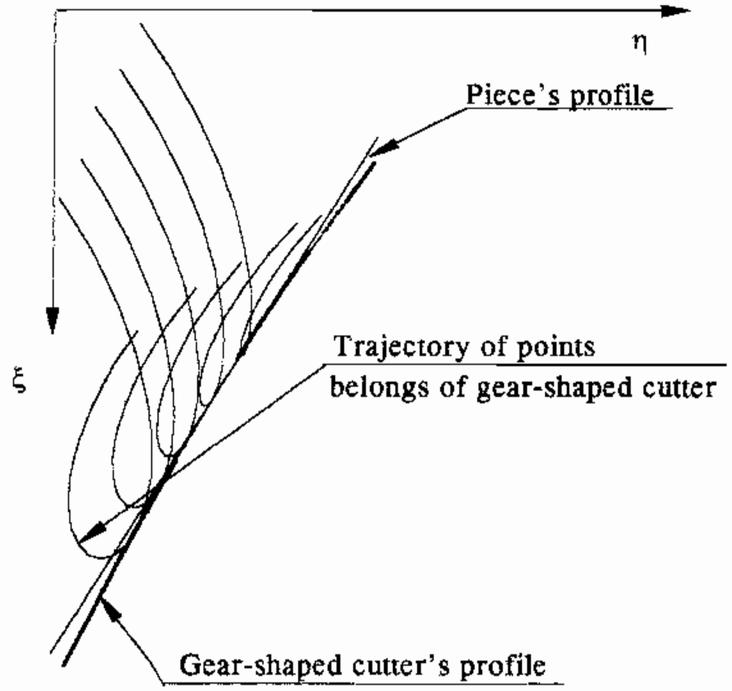


fig. 8. The profile of the gear-shaped cutter for internal chamfer grooves engendering.

4. Rotary cutter engendering.

It's suggest an example for application of the algorithm specific for rotary cutter engendering, at profiling tool for trapezoidal thread, figure 9.

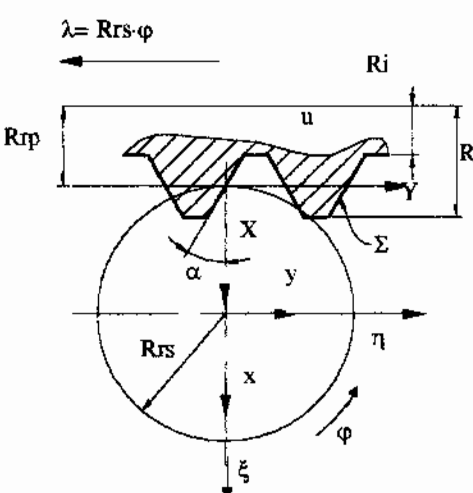


fig. 9. The trapezoidal threat; reference systems.

The process kinematics for engendering presume to know the relative motion.

$$\xi = \omega_3(\varphi)[X + a]. \quad (21)$$

Considerating the parametrical equations of the Σ profile;

$$X = -u \cdot \cos\alpha; \quad Y = u \cdot \sin\alpha, \quad (22)$$

with u as variable parameter is determined the trajectory family:

$$(T_\Sigma)_\varphi \begin{vmatrix} \xi = [-u \cdot \cos\alpha - Rrs]\cos\varphi + \\ +[u \cdot \sin\varphi - Rrs \cdot \varphi]\sin\varphi; \\ \eta = -[u \cdot \cos\alpha - Rrs]\sin\varphi + \\ +[u \cdot \sin\varphi - Rrs \cdot \varphi]\cos\varphi. \end{vmatrix} \quad (23)$$

The enveloping condition (4) presume the partial derived calculation:

$$\begin{aligned} &\xi'_u = \cos(\alpha + \varphi); \\ &\eta'_u = \sin(\alpha + \varphi); \\ &\xi'_\varphi = -[-u \cdot \cos\alpha - Rrs]\sin\varphi + \\ &+[u \cdot \sin\varphi - Rrs\varphi]\cos\varphi - \\ &- Rrs \cdot \sin\varphi; \\ &\eta'_\varphi = -[-u \cdot \cos\alpha - Rrs]\cos\varphi - \\ &-[u \cdot \sin\varphi - Rrs\varphi]\sin\varphi - \\ &- Rrs \cdot \cos\varphi. \end{aligned} \quad (24)$$

The (4) and (24) equation ensemble determine the trajectory family enveloping –the rotary cutter profile.

In figure 10 and table 5, it's shown the $(T_\Sigma)_\varphi$ trajectory (23), the rotary cutter profile and its coordinates.

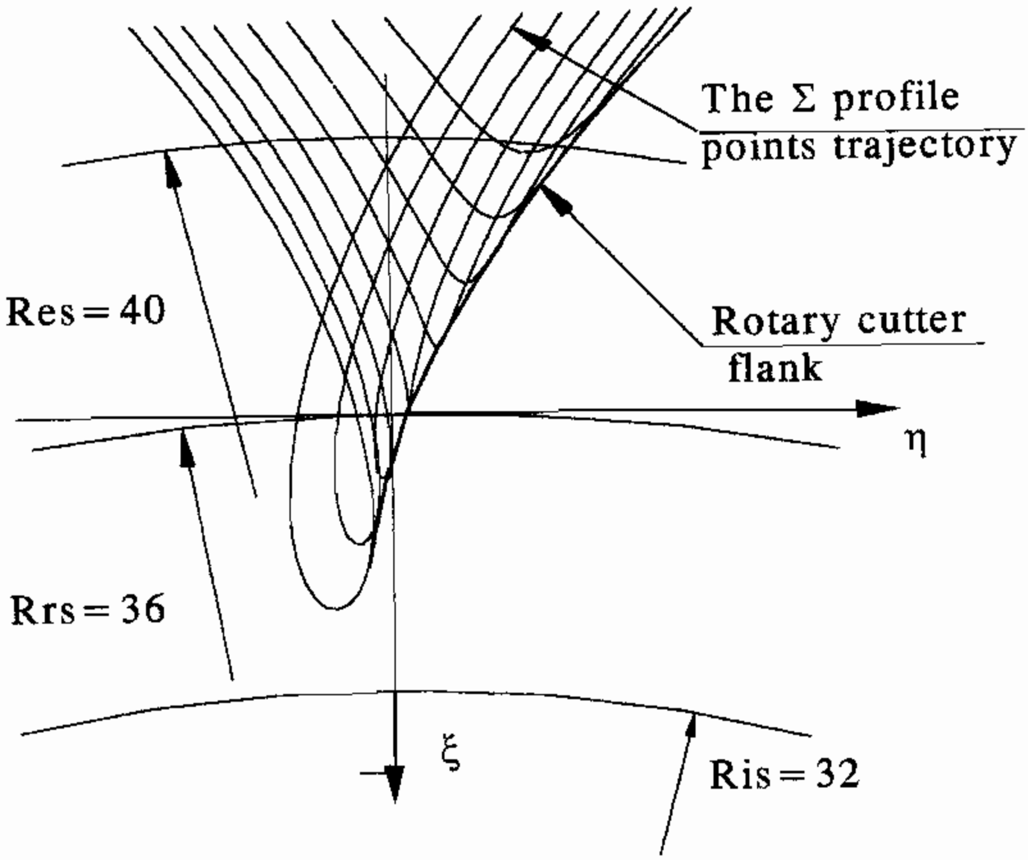


fig. 10. Rotary cutter profile.

Table 5.

Nr. crt.	ξ [mm]	η [mm]	u [mm]	Φ [rad]
1	-33.7798	-0.5061	-4.0	-0.318312
2	-33.9339	-0.5000	-3.5	-0.284398
3	-34.0674	-0.4884	-3.0	-0.243398
4	-34.2635	-0.4634	-2.5	-0.203398
5	-34.5063	-0.4224	-2.0	-0.162398
6	-34.7973	-0.3613	-1.51	-0.122398
7	-35.1617	-0.2690	-0.99	-0.0803982
8	-35.2381	-0.2477	-0.89	-0.0723982
9	-35.5448	-0.1553	-0.51	-0.0413982
10	-36.4922	0.19070	0.5	0.0406018
11	-36.6064	0.23830	0.61	0.0496018
12	-37.0168	0.41870	0.99	0.0806018
13	-37.6021	0.70180	1.5	0.121602
14	-38.2118	1.02800	2.0	0.161602
15	-38.8546	1.40610	2.5	0.201602
16	-39.5271	1.83900	3.0	0.241602
17	-40.2259	2.32970	3.5	0.281602
18	-40.9197	2.85880	4.0	0.318312

Programs algorithms

First the used layers are creating, and it is fixed a name for the output file.

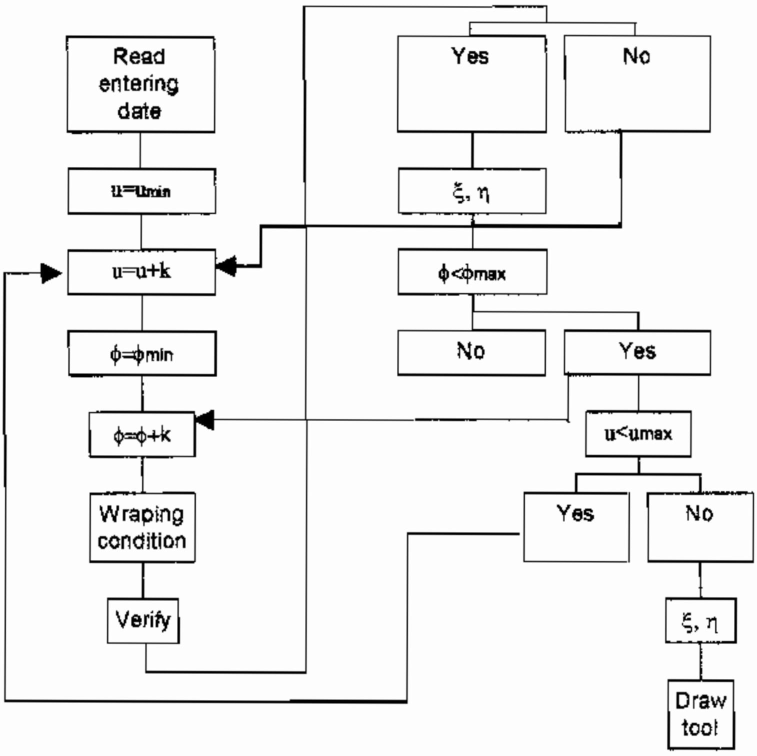
The user is asked for the entering date (dimensions, rolling radius, gearing ratio, the increments for u and φ, the admissible error value etc.) and the program stable the references systems.

The programs can now draw the piece.

Next the u and φ parameters will be calculated and a value for increment is attribuited. After this, will be calculated the values for ξ and η coordinates and the wrapping condition is verified. If the warping condition is accomplished, the coordinates will be writes in the output file. After each verification the parameters φ get a new values by adding the increment, upon the maximum value is reached. Next φ get the minimum value and the u parameter get a new value by adding the increment. The cycle is resumed by variation of the parameter φ.

Finally, when also the parameter φ reached the maximum value, the output file will be closed. Next the coordinates from this file are used to drawn the tool's profile.

The basic diagram of the program



Conclusions

The trajectory method, thus was presented, in principiale form and from showned examples is proved to be the capability to permit the approaching of the wrapping engendering problem for profiles associated of the rolling centroids pairs.

The method, analytical, have the advantage of a very simple application. More, the tracking of the trajectory, prevent the apparition of some rough errors.

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Metoda traiectoriilor cicloidale pentru studiul suprafeţelor în înfăşurare asociate unor centroide în rulare.
II Aplicaţii.
Rezumat.

În lucrare se prezintă câteva exemple de aplicare a teoremei metodei traiectoriilor la profilarea sculelor de tip cremalieră, cuţit-roată şi cuţit rotativ.

Se prezintă exemple numerice şi ilustrări grafice ale înfăşurătoarelor unei familii de traiectorii, pentru generarea unor profiluri simple ale semifabricatelor mărginite de vârtejuri de suprafeţe.

La méthode de les trajectoires pour l'étude de les surfaces en enveloppement associé a centroids en rouler.
II Aplicatons.
Resumé.

En c'est thése on présente quelques examples d'utilisation de la théoreme de la méthode de les trajectoires pour profiler des outils de la type crémaillére, couteau-roue et couteau rotatif.

Se présente dés examples noumériques et des illustration graphiques de les enveloppes d'un famille de trajectoires, pour engendrement quelques profiles simple des demi-produits limites de les treuils de surfaces.